Indian Statistical Institute, Bangalore

M.Math II Year, Second Semester

Mid-Sem Examination

Advanced Functional Analysis

Time: 3 hours — February 28, 2011 — Instructor: T.S.S.R.K.Rao

Total Score : $8 \times 5 = 40$

Answer all questions. Show all work. If you are assuming a 'Result' proved in class, state it correctly. It an answer is an immediate consequence of such a 'Result' , that 'Result' also needs to be proved.

1. Let X be a topological vector space. $A \subset X$ is said to be totally bounded, if for any neighborhood $\cup$ of 0, there exists a finite set $F \subset A$ such that $A \subset (F + \cup)$. Show that any totally bounded set is bounded and any compact set is bounded.

2. Consider $C[0,1]$. Show that there is a locally convex topological vector space topology on $C[0,1]$ that is different from the norm topology. Give all the details of your answer.

3. Consider $l^2 = \{\{\alpha_n\}_{n\geq 1} : \Sigma \mid \alpha_n \mid^2 \ < \infty)\}$. Let $U \subset l^2$ be a proper neighborhood of 0. Show that there exists a sequence $x = \{\alpha_n\}_{n\leq 1} \ \varepsilon \ U$ with infinitely many non-zero coordinates.

4. Let $c_0 = \{\{\alpha_n\}_{n\leq 1} : \lim \alpha_n = 0\}$. Let d be an invariant matric on c_0 making it a topological vector space. Let d' denote the supremum metric on c_0. Suppose that topologies generated by d and d' are the same Show that d is a complete metric.

5. Let $l^1 = \{\{\alpha_n\}_{n\leq 1} : \Sigma \mid \alpha_n \mid < \infty\}$ with the usual norm. Show that this space is separable when equipped with the weak topology.

6. Consider $L^1[0,1]$. Let X be the set of all polynomials in $L^1[0,1]$, with the usual norm. Give an example if a weak$^*-$ bounded set in X^* that is not bounded. Give complete details of your answer.

7. Let X be a topological vector space with a, balanced and bounded neighborhood of 0. Show that X is metrizable (you may assume that metrization theorem).

8. On the space $C_b(\mathbb{R})$ of bounded continuous functions, consider the sequence of semi -norms, $p_n(f) = \sup\limits_{[-n,n]} \mid f \mid$. Describe convergent sequences and bounded sets in the topology generated by this family $\{p_n\}_{n\leq 1}$.